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Choquet Order and Jordan Maps

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1. INTRODUCTION

The aim of this paper is to continue our investigation initiated in [7] and to bring several new aspects of the interaction between Choquet theory and theory of state spaces of operator algebras. Our previous research has shown that there is a connection between the Choquet order of orthogonal measures and posets of abelian subalgebras of von Neumann algebras. Utilizing this relationship we demonstrated that any isomorphism between the order set of orthogonal measures with a fixed barycenter, endowed with the Choquet order, is induced by a Jordan $*$-isomorphisms of certain associated von Neumann algebras. The isomorphisms of the whole posets of orthogonal measures were treated in [8]. In this note we shall consider more general situation when states and their decompositions are invariant with respect to an action of a discrete automorphisms group of a given C^*-algebra. In this situation, we identify Choquet order with set theoretic order of abelian von Neumann subalgebras associated with covariant GNS representation. We also provide discrete version of this result for orthogonal decompositions of invariant states. Besides we provide ramifications of some results in [7, 8].

Let us remark that concept of Choquet order is crucial for decomposition theory of states and representations [1]. The reason is that maximal elements in this order provide decomposition of a state into integral mixture of extremal points. If we consider invariant states with respect to a given dynamical system then maximal elements in Choquet order induce decompositions of state into ergodic states that correspond to pure phases of quantum physical system. It is a motivation for our research.

2. CHOQUET ORDER AND ABSTRACT COMPACT CONVEX SETS

In this part we review basic facts on Choquet theory on abstract compact convex sets, including recent results and some new aspects.

We start with summarizing basic facts on ordered sets. Let $(X, \leq)$ be a *partially ordered set* (a *poset* for short). Suppose that X has the least element 0. An element x of X is called *atom* if it is nonzero and $0 \leq y \leq x$ implies $y = x$ or $y = 0$. A map $F : X \to Y$ between two posets $(X, \leq)$ and $(Y, \leq)$ is called an *order isomorphism* if it is a bijection preserving order in both directions, that is

$$a \leq b \iff F(a) \leq F(b) \quad \text{for all } a, b \in X.$$

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